Introduction

Volatility in equity market has become a matter of mutual concern in recent years for investors, regulators and brokers. Stock return volatility hinders economic performance through consumer spending\(^1\). Stock Return Volatility may also affect business investment spending\(^2\). Further the extreme volatility could disrupt the smooth functioning of the financial system and lead to structural or regulatory changes.

Volatility of stock returns in the developed countries has been studied extensively. After the seminal work of Engle(1982) on Autoregressive Conditional Heteroscedasticity (ARCH) model on UK inflation data and its Generalized form GARCH(Generalized ARCH) by Bollerslev (1986), much of the empirical work used these models and their extensions ( See French, Schwert and Stambaugh 1987, Akgiray 1989, Schwert, 1990, Chorhay and Tourani,1994, Andersen and Bollerslev, 1998) to model characteristics of financial time series.

Starting with the pioneering work of Mandelbrot (1963) and Fama (1965), various features of stock returns have been extensively documented in the literature which are important in modeling stock market volatility. It has been found that

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\(^1\) Garmer A.C., 1988, Has Stock Market Crash Reduced Customer Spending? Economic Review, Federal Reserve Bank of Kansas City, April, 3-16.

stock market volatility is time varying and it also exhibits positive serial correlation (volatility clustering). This implies that changes in volatility are non-random. Moreover, the volatility of returns can be characterized as a long-memory process as it tends to persist (Bollerslev, Chou and Kroner, 1992). Schwert (1989) agreed with this argument. Fama (1965) also found the similar evidence. Baillie and Bollerslev (1991) observed that the volatility is predictable in the sense that it is typically higher at the beginning and at the close of trading period. Akgiray (1989) found that GARCH (1, 1) had better explanatory power to predict future volatility in US stock market. Poshakwale and Murinde (2001) modeled volatility in stock markets of Hungary and Poland using daily indexes. They found that GARCH(1,1) accounted for nonlinearity and volatility clustering. Poon and Granger (2003) provided comprehensive review on volatility forecasting. They examined the methodologies and empirical findings of 93 research papers and provided synoptic view of the volatility literature on forecasting. They found that ARCH and GARCH classes of time series models are very useful in measuring and forecasting volatility.


Balaban, Bayar and Faff (2002) investigated the forecasting performance of both ARCH-type models and non-ARCH models applied to 14 different countries. They observed that non-ARCH models usually produce better forecast than ARCH type models. Finally, Exponential GARCH is the best among ARCH-type models. Pan and Zhang (2006) use Moving Average, Historical Mean, Random Walk, GARCH, GJR-GARCH, EGARCH and APARCH to forecast volatility of two Chinese Stock Market indices; Shanghai and Shenzhen. The study found that Among GARCH models, GJR-GARCH and EGARCH outperforms other ARCH models for Shenzhen stock market. Magnus and Fosu (2007) employed Random Walk, GARCH(1,1), TGARCH(1,1) and EGARCH(1,1) to forecast Ghana Stock Exchange. GARCH(1,1) provides the best forecast according to three different criterias out of four. On the other hand, EGARCH and Random Walk produces the worst forecast.

Foregoing discussion suggests that the modeling of the stock markets volatility and its forecasting is of great importance to academics, policy makers, and financial markets participants. Predicting volatility might enable one to take risk-free decision making including portfolio selection and option pricing. High levels of volatility in a stock market can lead to a general erosion of investors’ confidence and an outflow of capital from stock markets, volatility has become a matter of mutual concern for
government, management, brokers and investors. It is therefore necessary for us to explore stock market volatility and also identify a model that gives better prediction. The rest of the paper is organized as follows. Section II provides research design used in the study. Empirical results are discussed in Section III. Section IV summarizes.

**Research Design**

**Period of study**

We collected data on daily closing price Banking index namely Sensex of Bombay Stock Exchange from January 1, 2009 to June 24, 2014. It consists of 1359 observations. Banking sector reforms such as fall in interest rates, and enactment of Securitization Bill have given a major fillip to Indian banking industry. These developments have significantly impacted the performance of bank stocks and bank stocks have emerged as a major segment in the equity markets.

The period of the study is the most recent one. These stock markets have become increasingly integrated. The trades between countries have increased. They are playing an important role in the world economy. These might have influenced the behavior and the pattern of volatility and therefore it will be instructive to study volatility in this period.

**Methodology**

Daily returns are identified as the difference in the natural logarithm of the closing index value for the two consecutive trading days. Volatility is defined as;

\[
\sigma = \sqrt{\frac{1}{n-1} \sum_{i=1}^{n} (R_i - \bar{R})^2}
\]  

**Equation 1**

where \( \bar{R} \) = Average return (logarithmic difference) in the sample.

In comparing the performance of linear model with its nonlinear counterparts, we first used ARIMA\(^3\) models. Nelson (1990b) explains that the specification of mean equation bears a little impact on ARCH models when estimated in continuous time. Several studies recommend that the results can be extended to discrete time. We follow a classical approach of assuming the first order autoregressive structure for conditional mean as follows:

\[
R_t = a_0 + a_1 R_{t-1} + \epsilon_t
\]  

**Equation 2**

where \( R_t \) is a stock return, \( a_0 + a_1 R_{t-1} \) is a conditional mean and \( \epsilon_t \) is the error term in period t. The error term is further defined as:

\[
\epsilon_t = \nu_t \sigma_t
\]  

**Equation 3**

where \( \nu_t \) is white noise process that is independent of past realizations of \( \epsilon_{t-i} \). It has zero mean and standard deviation of one. In the context of Box and Jenkins (1976), the series should be stationary before ARIMA models are used. Therefore, Augmented Dickey Fuller test (ADF) is used to test for stationarity of the return series. It is a test for detecting the presence of stationarity in the series. The early and pioneering work on testing for a unit root in time series was done by Dickey and Fuller (1979 and 1981). If the variables in the regression model are not stationary, then it can be shown that the standard assumptions for asymptotic analysis

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\(^3\) A process that combines Autoregressive process (AR) and Moving Average terms (MA) terms. AR process where the present observations depend on the previous observations and MA is a weighted average of the present and the recent past observations of a process.
ADF tests for a unit root in the univariate representation of time series. For a return series $R_t$, the ADF test consists of a regression of the first difference of the series against the series lagged $k$ times as follows:

$$\Delta r_t = \alpha + \delta r_{t-1} + \sum_{i=1}^{p} \beta_i \Delta r_{t-i} + \epsilon_t \hspace{1cm} \text{Equation 4}$$

The null hypothesis is $H_0: \delta = 0$ and $H_1: \delta < 1$. The acceptance of null hypothesis implies nonstationarity. We can transform the nonstationary time series to stationary time series either by differencing or by detrending. The transformation depends upon whether the series is difference stationary or trend stationary.

One needs to specify the form of the second moment, variance, $\sigma_t^2$ for estimation. ARCH and GARCH models assume conditional heteroscedasticity with homoscedastic unconditional error variance. That is, the changes in variance are a function of the realizations of preceding errors and these changes represent temporary and random departure from a constant unconditional variance.

The advantage of GARCH model is that it captures the tendency in financial data for volatility clustering. It, therefore, enables us to make the connection between information and volatility explicit since any change in the rate of information arrival to the market will change the volatility in the market. In empirical applications, it is often difficult to estimate models with large number of parameters, say ARCH ($q$). To circumvent this problem, Bollerslev (1986) proposed GARCH ($p$, $q$) models. The conditional variance of the GARCH ($p$, $q$) process is specified as

$$h_t = \alpha_0 + \sum_{j=1}^{q} \alpha_j \epsilon_{t-j}^2 + \sum_{i=1}^{p} \beta_i h_{t-i} \hspace{1cm} \text{Equation 5}$$

with $\alpha_0 > 0$, $\alpha_1$, $\alpha_2$, $\ldots$, $\alpha_q > 0$ and $\beta_1$, $\beta_2$, $\beta_3$, $\ldots$, $\beta_p > 0$ to ensure that conditional variance is positive. In GARCH process, unexpected returns of the same magnitude (irrespective of their sign) produce same amount of volatility. The large GARCH lag coefficients $\beta_i$ indicate that shocks to conditional variance takes a long time to die out, so volatility is ‘persistent.’ Large GARCH error coefficient $\alpha_i$ means that volatility reacts quite intensely to market movements and so if $\alpha_i$ is relatively high and $\beta_i$ is relatively low, then volatilities tend to be ‘spiky’. If $(\alpha + \beta)$ is close to unity, then a shock at time $t$ will persist for many future periods. A high value of it implies a ‘long memory.’

**EGARCH Model**

GARCH models successfully capture thick tailed returns, and volatility clustering, but they are not well suited to capture the “leverage effect” since the conditional variance is a function only of the magnitudes of the lagged residuals and not their signs. In the exponential GARCH (EGARCH) model of Nelson (1991) $\sigma_t^2$ depends upon the size and the sign of lagged residuals. The specification for the conditional variance is:

$$\log(\sigma_t^2) = \alpha_0 + \sum_{j=1}^{q} \beta_j \log(\sigma_{t-j}^2) + \sum_{i=1}^{p} \alpha_i \frac{\epsilon_{t-i}}{\sigma_{t-i}} + \sum_{h=1}^{r} \beta_h \frac{\epsilon_{t-h}}{\sigma_{t-h}} \hspace{1cm} \text{Equation 6}$$

Note that the left-hand side is the log of the conditional variance. This implies that the leverage effect is exponential, rather than quadratic, and that forecasts of the conditional variance are guaranteed to be nonnegative thus eliminating the need for parameter restrictions to impose non-
negativity as in the case of ARCH and GARCH models. The presence of leverage effects can be tested by the hypothesis that $\gamma_h < 0$. The impact is asymmetric if $\gamma_h \neq 0$.

**TGARCH Model**

In ARCH / GARCH models both positive and negative shocks of same magnitude will have exactly same effect in the volatility of the series. T-GARCH model helps in overcoming this restriction. TARCH or Threshold GARCH model was introduced independently by Zakoin (1994) and Glosten, Jaganathan and Runkle (1993). The generalized specification for the conditional variance is given by:

$$
\sigma_t^2 = \alpha + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2 + \sum_{i=1}^{p} \alpha_i \varepsilon_{t-i}^2 + \sum_{h=1}^{\infty} \gamma_h \varepsilon_{t-h}^2 \cdot d_{t-h}
$$

**Equation 7**

Where $d_t = 1$ if $\varepsilon_t < 0$ and zero otherwise.

In this model, good news, $\varepsilon_{t-i} > 0$, and bad news, $\varepsilon_{t-i} < 0$, have differential effect on the conditional variance; good news has an impact of $\alpha_i$, while bad news has an impact of $\alpha_i + \gamma_i$. If $\gamma_i > 0$, bad news increases volatility, and we say that there is a leverage effect for the i-th order. If $\gamma_i \neq 0$, the news impact is asymmetric. The main target of this model is to capture asymmetries in terms of positive and negative shocks.

**Forecasting Evaluation**

Root mean squared error (RMSE), mean absolute error (MAE), mean absolute percentage error (MAPE) and Theil inequality coefficient (TIC) are employed to measure the accuracy of the forecasting models.

$\text{RMSE} = \sqrt{\frac{\sum_{t=184}^{365} (\sigma_{a,t} - \sigma_{f,t})^2}{182}}$

$\text{MAE} = \frac{\sum_{t=184}^{365} |\sigma_{a,t} - \sigma_{f,t}|}{182}$

$\text{MAPE} = 100 \frac{\sum_{t=184}^{365} |\sigma_{a,t} - \sigma_{f,t}|}{\sum_{t=184}^{365} \sigma_{a,t}}$

$\text{TIC} = \sqrt{\frac{\sum_{t=184}^{365} (\sigma_{a,t} - \sigma_{f,t})^2}{181}} - \sqrt{\frac{\sum_{t=184}^{365} \sigma_{a,t}^2}{182}} + \sqrt{\frac{\sum_{t=184}^{365} \sigma_{f,t}^2}{182}}$

Where $\sigma_{a,t}$ is the actual volatility and $\sigma_{f,t}$ is the forecasted volatility. The model with better forecasting power has lower values of all the above measures compare to other models.

**Empirical results**

The descriptive statistics for the return series include mean, standard deviation, skewness, kurtosis,Jarque-Bera and Ljung Box. ARCH-LM statistics are also exhibited in the Table 1.

**Table 1**: Descriptive Statistics of Daily Returns

<table>
<thead>
<tr>
<th>Statistic</th>
<th>Sensex</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>0.00071</td>
</tr>
<tr>
<td>Standard deviation</td>
<td>0.01342</td>
</tr>
<tr>
<td>Skewness</td>
<td>1.14196</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>18.95906</td>
</tr>
<tr>
<td>Jarque-Bera Statistics</td>
<td>14706.5(0.000)</td>
</tr>
<tr>
<td>Q^2(12)</td>
<td>62.96(0.000)</td>
</tr>
<tr>
<td>ARCH LM statistics ( at Lag =1)</td>
<td>1.09(0.29)</td>
</tr>
<tr>
<td>ARCH LM statistics ( at Lag =5)</td>
<td>11.59(0.041)</td>
</tr>
</tbody>
</table>
Notes: ARCH LM statistic is the Lagrange multiplier test statistic for the presence of ARCH effect. Under null hypothesis of no heteroscedasticity, it is distributed as $\chi^2(k)$. $Q^2(K)$ is the Ljung Box statistic identifying the presence of autocorrelation in the squared returns. Under the null hypothesis of no autocorrelation, it is distributed as $\chi^2(k)$.

The mean returns for all the stock indices are very close to zero indicating that the series are mean reverting. The return distribution is negatively skewed, indicating that the distribution is non-symmetric. Large value of Kurtosis suggests that the underlying data are leptokurtic or thick tailed and sharply peaked about the mean when compared with the normal distribution. Since GARCH model can feature this property of leptokurtosis evidence in the data.

The Jarque-Bera\(^4\) statistics calculated and reported in the Table-1 to test the assumption of normality. The results show that the null hypothesis of normality in case of both the stock markets is rejected.

The Ljung-Box LB\(^2\) (12) statistical values of all the series respectively rejects significantly the zero correlation null hypothesis. It suggests that there is a clustering of variance. Thus, the distribution of square returns depends on current square returns as well as several periods’ square returns, which will result in volatility clustering.

Stationarity condition of the Sensexdaily return series were tested by Augmented Dickey-Fuller Test (ADF). The results of this test are reported in the Table2.

\(^4\) The B-J test statistic is $T[\text{skewness}^2/6+\text{(kurtosis-3)}^2/24]$. 

<table>
<thead>
<tr>
<th>Null Hypothesis</th>
<th>Test statistics</th>
<th>Mackinnon Asymptotic</th>
</tr>
</thead>
<tbody>
<tr>
<td>Presence of Unit root</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Level</td>
<td>-2.26 (0.19)</td>
<td>-34.60 (0.00)</td>
</tr>
<tr>
<td>First Difference</td>
<td>-34.60 (0.00)</td>
<td>-3.44</td>
</tr>
<tr>
<td>Sensex</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Intercept</td>
<td>-2.55 (0.30)</td>
<td>-34.59 (0.00)</td>
</tr>
<tr>
<td>Trend &amp; Intercept</td>
<td>-34.59 (0.00)</td>
<td>-3.97</td>
</tr>
<tr>
<td>Trend coefficient</td>
<td>0.00 (0.18)</td>
<td>-0.00 (0.62)</td>
</tr>
<tr>
<td>None</td>
<td>1.89 (0.98)</td>
<td>-34.52 (0.00)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-2.57</td>
</tr>
</tbody>
</table>

ADF statistics in level series shows presence of unit root in the stock markets as their Mackinnon’s value do not exceed the critical value at 1% level. It suggests that the price series is nonstationary. The trend coefficients of the series is statistically insignificant suggesting absence of any trend in stock market. It is, therefore, necessary to transform the series to make it stationary by taking its first difference. ADF statistics reported in the Table 2 show that the null hypothesis of a unit root is rejected. The absolute computed values for the index is higher than the MacKinnon critical value at 1% level. Thus, the results of the indices show that the first difference series is stationary.

To test for heteroscedasticity, the ARCH-LM test is applied to the series. The results are reported in Table 1. The ARCH-LM test at lag length 1 and 5 indicate presence of ARCH effect in the residuals in both the stock markets. It implies clustering of volatility where large changes tend to be
followed by large changes, of either sign and small changes tend to be followed by small changes (Engle, 1982 and Bollerslev, 1986). The Conditional volatility of returns may not only be dependent on the magnitude of error terms but also on its sign. We checked for asymmetry in both the stock markets using EGARCH and TARCH models. The results are presented in the Table 3.

Table 3 Coefficients of Asymmetric Models

<table>
<thead>
<tr>
<th>Sensex</th>
<th>EGARCH(1,1)</th>
<th>TARCH(1,1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_0$</td>
<td>-0.5952 (0.000)</td>
<td>0.0000 (0.000)</td>
</tr>
<tr>
<td>$\alpha_1$</td>
<td>0.2070(0.000)</td>
<td>0.0396 (0.000)</td>
</tr>
<tr>
<td>$\beta_1$</td>
<td>0.9726 (0.000)</td>
<td>0.8792 (0.000)</td>
</tr>
<tr>
<td>$\alpha_1 + \beta_1$</td>
<td>-0.0908 (0.005)</td>
<td>0.1262 (0.005)</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>(RESID(-1)^2)*RESID(-1)&lt;0)</td>
<td></td>
</tr>
<tr>
<td>SQRT(GARCH)</td>
<td>Log liklihood</td>
<td>7620.655 7627.577</td>
</tr>
<tr>
<td>AIC</td>
<td>5.829751</td>
<td>5.835052</td>
</tr>
<tr>
<td>SBC</td>
<td>5.814026</td>
<td>5.819326</td>
</tr>
<tr>
<td>ARCH-LM(5) Test</td>
<td>3.207 (0.668)</td>
<td>5.13 (0.40)</td>
</tr>
</tbody>
</table>

Table 4 Volatility Forecasting Evaluation

<table>
<thead>
<tr>
<th>Model</th>
<th>RMSE</th>
<th>MAE</th>
<th>MAPE</th>
<th>TIC</th>
</tr>
</thead>
<tbody>
<tr>
<td>GARCH (1,1)</td>
<td>1.217</td>
<td>0.9302</td>
<td>0.0000016</td>
<td>0.5017</td>
</tr>
<tr>
<td>EGARCH (1,1)</td>
<td>1.614</td>
<td>1.432</td>
<td>0.000025</td>
<td>0.5182</td>
</tr>
<tr>
<td>GJR-GARCH (1,1)</td>
<td>2.871</td>
<td>1.497</td>
<td>0.000026</td>
<td>0.5235</td>
</tr>
</tbody>
</table>
Table 2 gives the actual forecast error statistics for each model. In the case of RMSE, exponential GARCH provides the best volatility forecast. If we look at MAE and MAPE, GARCH(1,1) models provide better forecasting than other models.

The Theil Inequality Coefficient (TIC) is a scale invariant measure that always lies between Zero and one, where Zero indicates a perfect fit. Looking at this coefficient we can say that GARCH(1,1) model is the best forecasting model. It is interesting to note that Exponential and Threshold GARCH do not provide better forecast than GARCH model. All the forecasting measures hints at GARCH(1,1) model for better forecasting of conditional volatility.

The volatility in the Sensex exhibits the persistence of volatility, mean reverting behavior and volatility clustering. Various diagnostic tests indicate volatility clustering and the response to news arrival is asymmetrical, meaning that impact of good and bad news is not the same. By the application of asymmetrical GARCH models like EGARCH and TARCH, we conclude that there is a presence of leverage effect in both the stock markets in India. These models suggest that the volatility appears to be more when price decline than when price increases.

We employed three different models to forecast volatility; GARCH(1,1), EGARCH(1,1) and GJR-GARCH(1,1). We used RMSE, MAE, MAPE and TIC to check forecasting accuracy. Our results indicate that GARCH (1,1) is the best forecasting model.

References


